Trendline and Regression Analysis (Part 1):

Simple Linear Regression

BM 4419

Business Analytics

Introduction to Regression Analysis

DCOVA

- Regression analysis is used to:
  - Predict the value of a dependent variable based on the value of at least one independent variable
  - Explain the impact of changes in an independent variable on the dependent variable

Dependent variable: the variable we wish to predict or explain

Independent variable: the variable used to predict or explain the dependent variable

Simple Linear Regression Model

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- Only **one** independent variable, X
- Relationship between X and Y is described by a linear function
- Changes in Y are assumed to be related to changes in X

Types of Relationships

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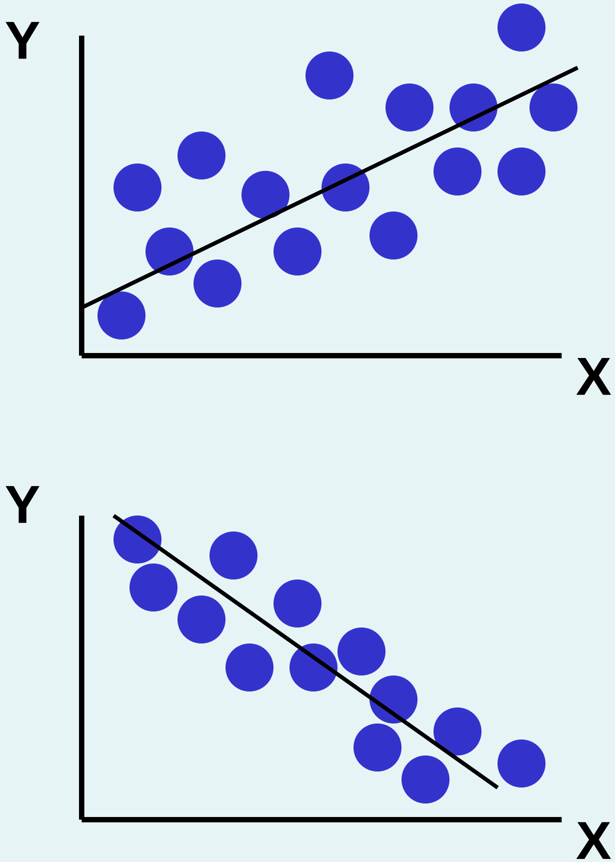
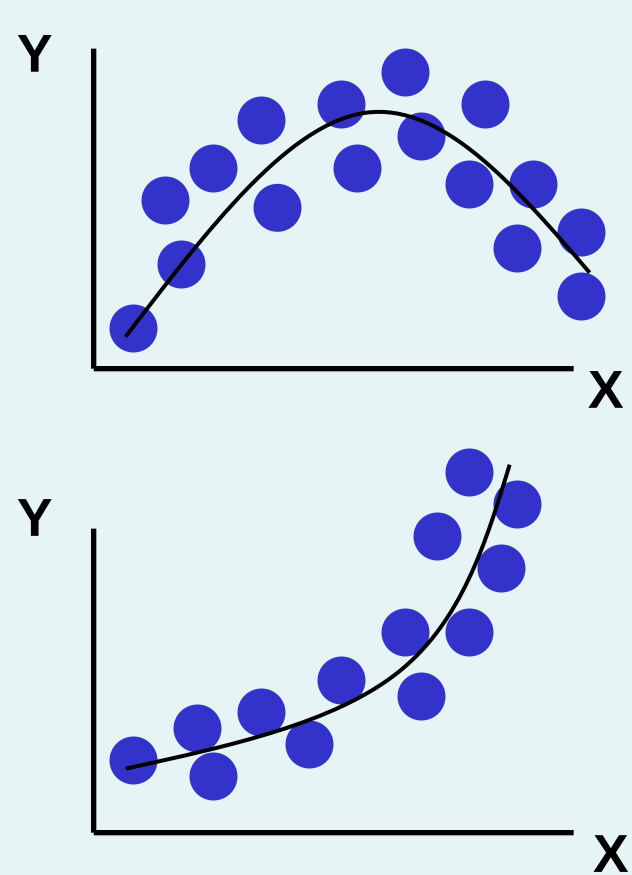
Linear relationships



Curvilinear relationships



Types of Relationships

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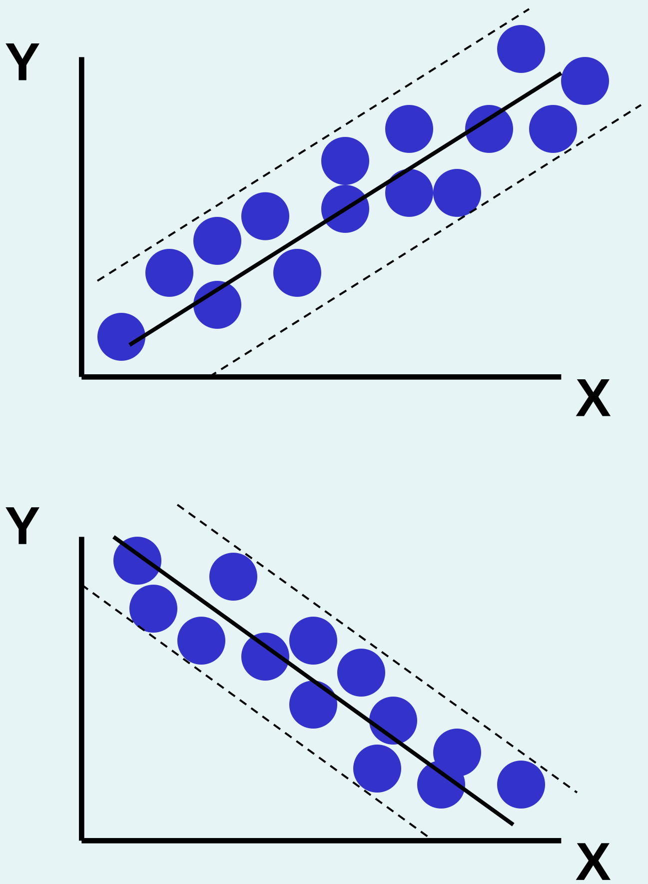
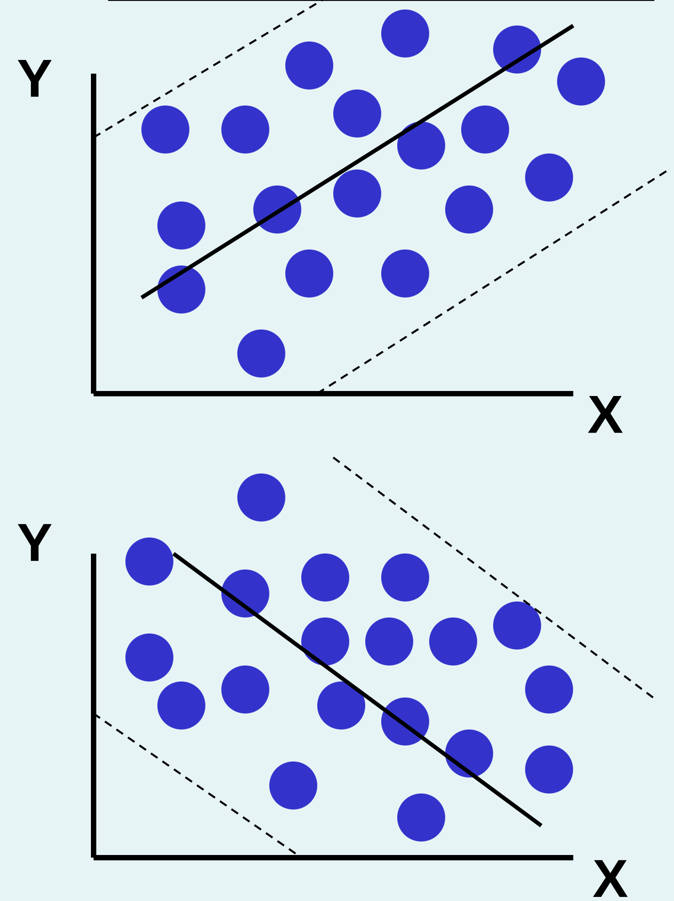
Strong relationships



Weak relationships



Types of Relationships

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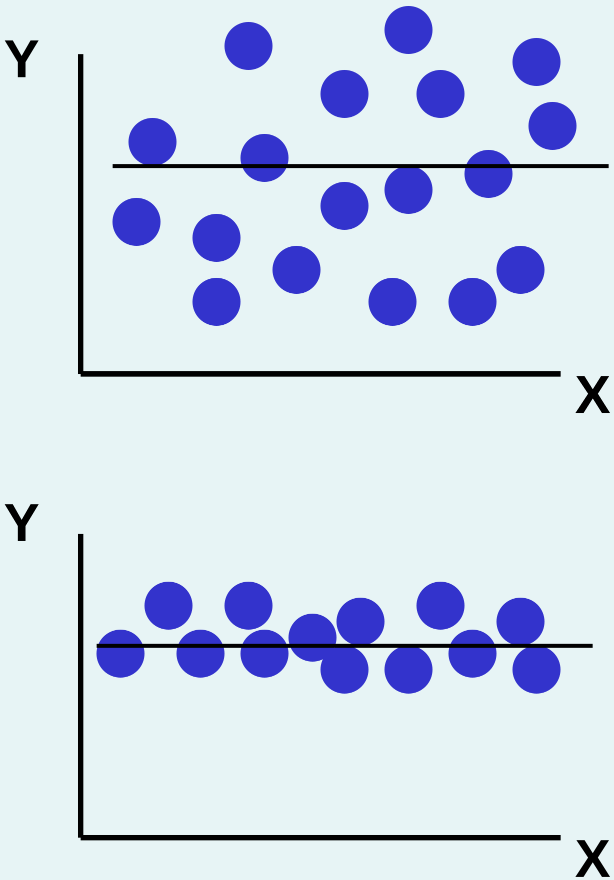
(continued)

No relationship



Simple Linear Regression Model

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Dependent Variable

Population Y intercept

Population Slope Coefficient

Independent Variable

Random Error term

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$$

Linear component

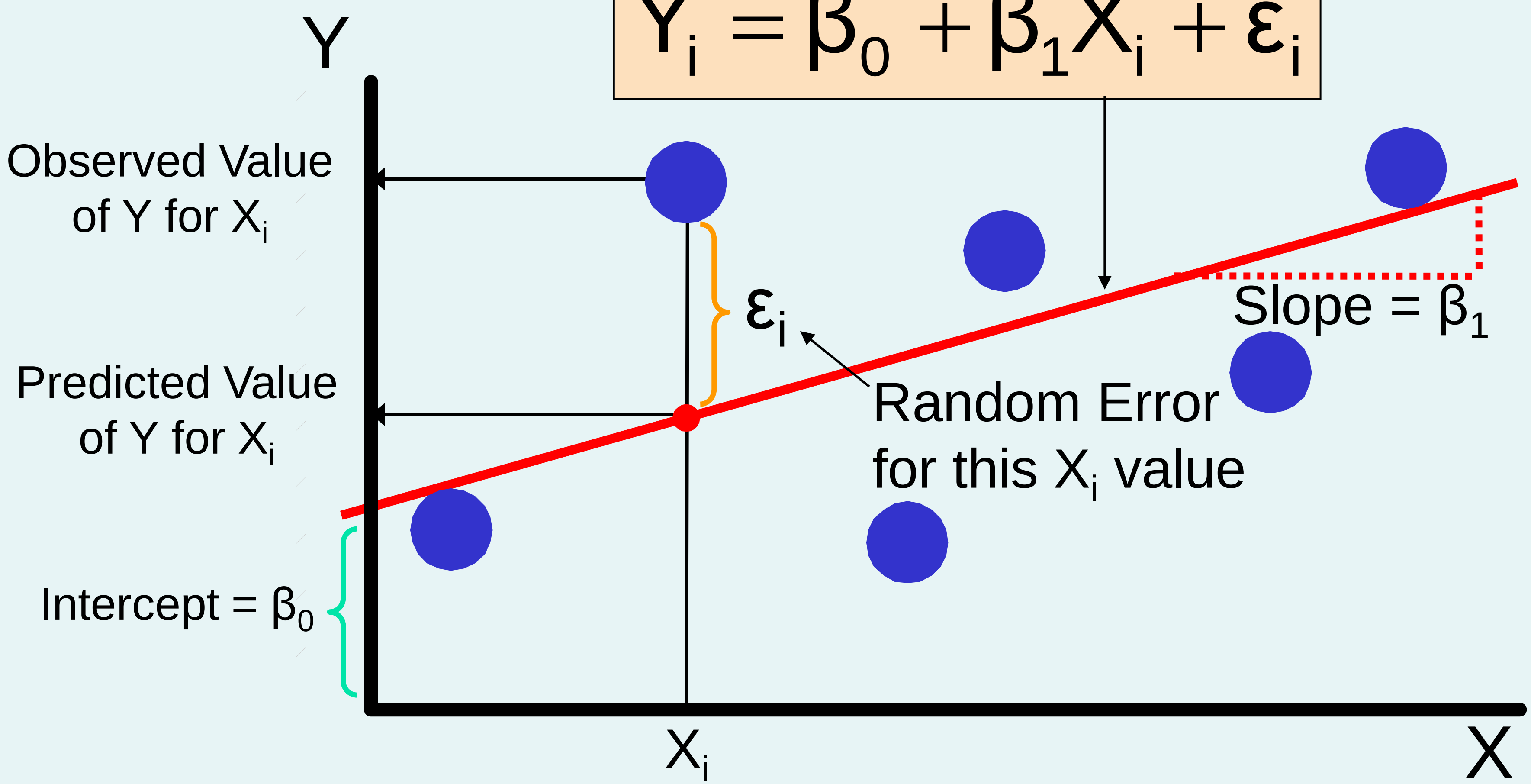
Random Error component

Simple Linear Regression Model

DCOVA

(continued)

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$$



Simple Linear Regression Equation (Prediction Line)

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The simple linear regression equation provides an estimate of the population regression line

$$\hat{Y}_i = b_0 + b_1 X_i$$

- $\hat{Y}_i$: Estimated (or predicted) Y value for observation i
- b_0: Estimate of the regression intercept
- b_1: Estimate of the regression slope
- X_i: Value of X for observation i

Interpretation of the Slope and the Intercept

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- b_0 is the estimated average value of Y when the value of X is zero

- b_1 is the estimated change in the average value of Y as a result of a one-unit increase in X

Simple Linear Regression Example

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- A real estate agent wishes to examine the relationship between the selling price of a home and its size (measured in square feet)

- A random sample of 10 houses is selected
  - Dependent variable (Y) = house price in $1000s
  - Independent variable (X) = square feet

Simple Linear Regression Example: Data

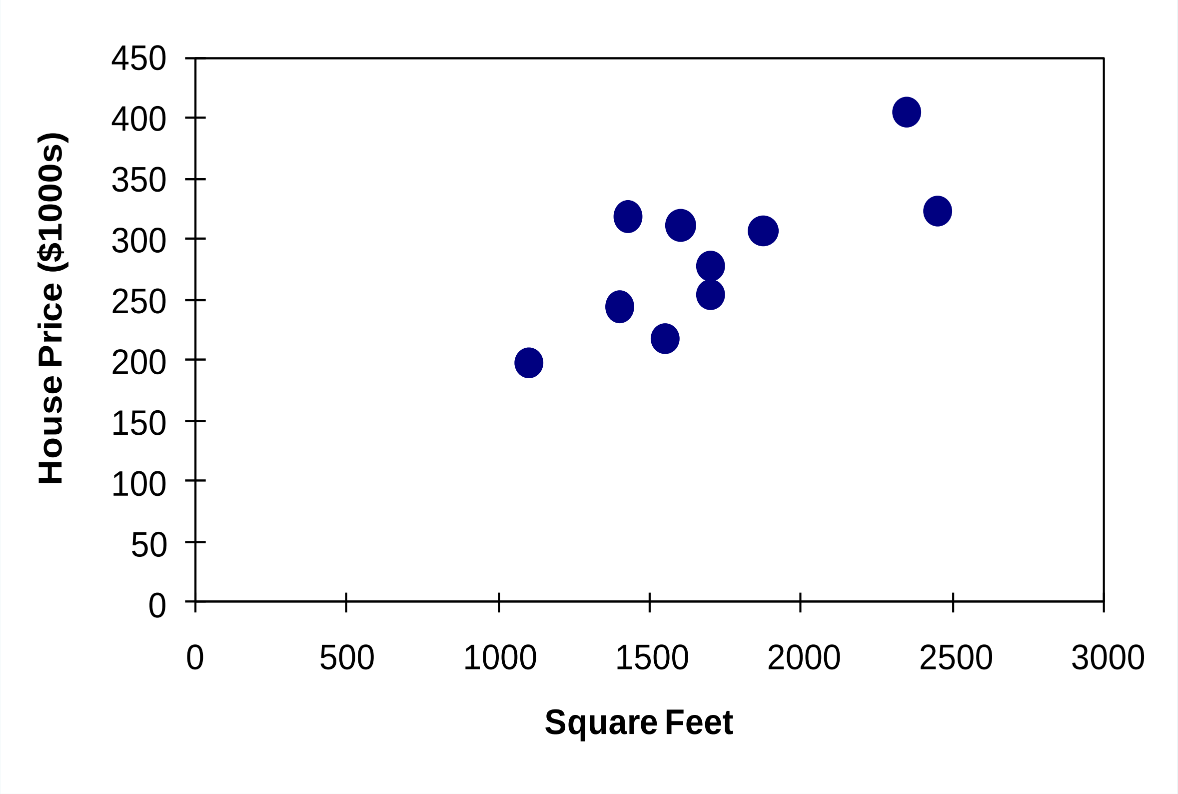
DCOVA

House Price in $1000s (Y)	Square Feet (X)
245	1400
312	1600
279	1700
308	1875
199	1100
219	1550
405	2350
324	2450
319	1425
255	1700

Simple Linear Regression Example: Scatter Plot

DCOVA

House price model: Scatter Plot



Simple Linear Regression Example: Using Excel Data Analysis Function

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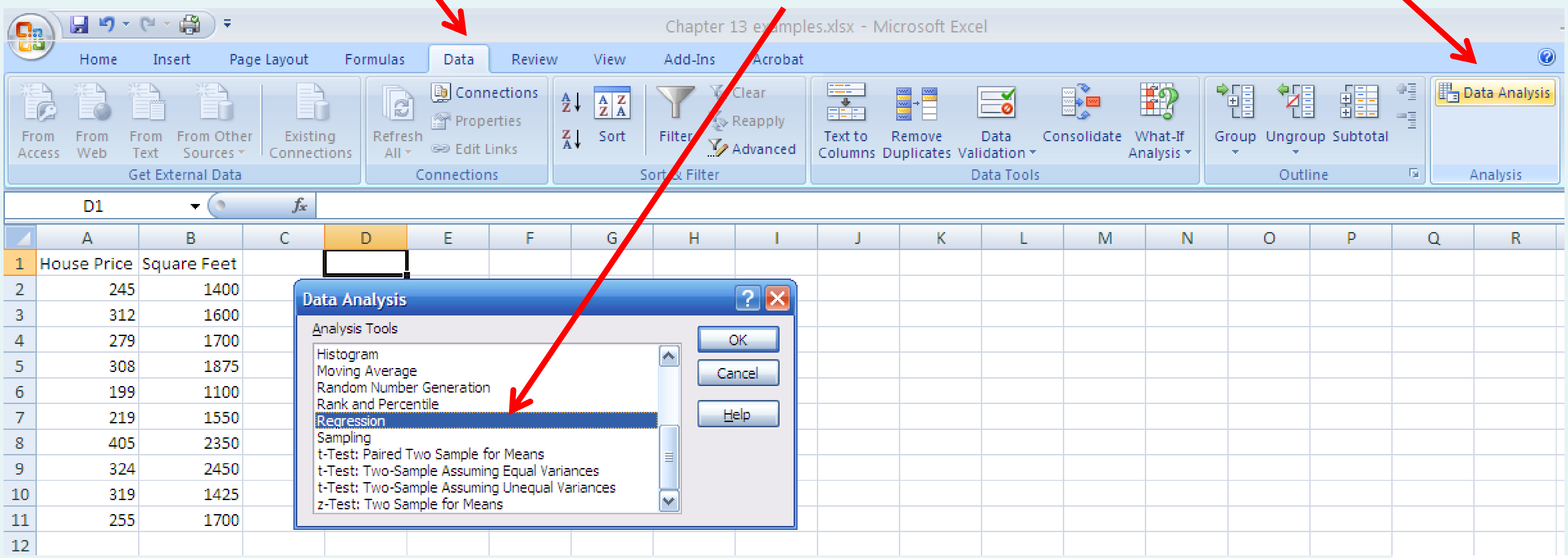
1. Choose Data

2. Choose Data Analysis

3. Choose Regression



House Price	Square Feet
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324	2450
319	1425
255	1700

Simple Linear Regression Example: Using Excel Data Analysis Function

(continued)

<u>Enter Y's and X's and desired options</u>

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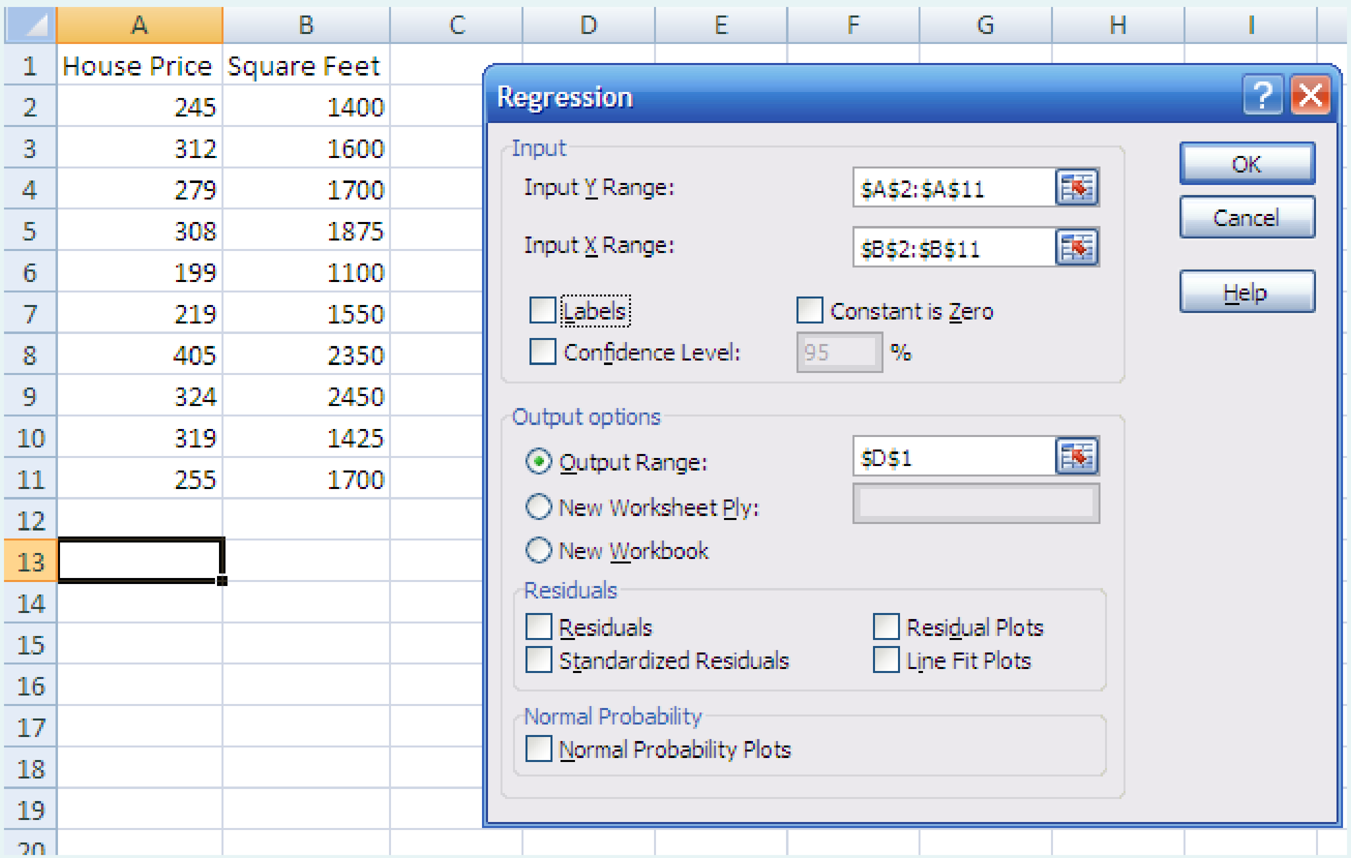
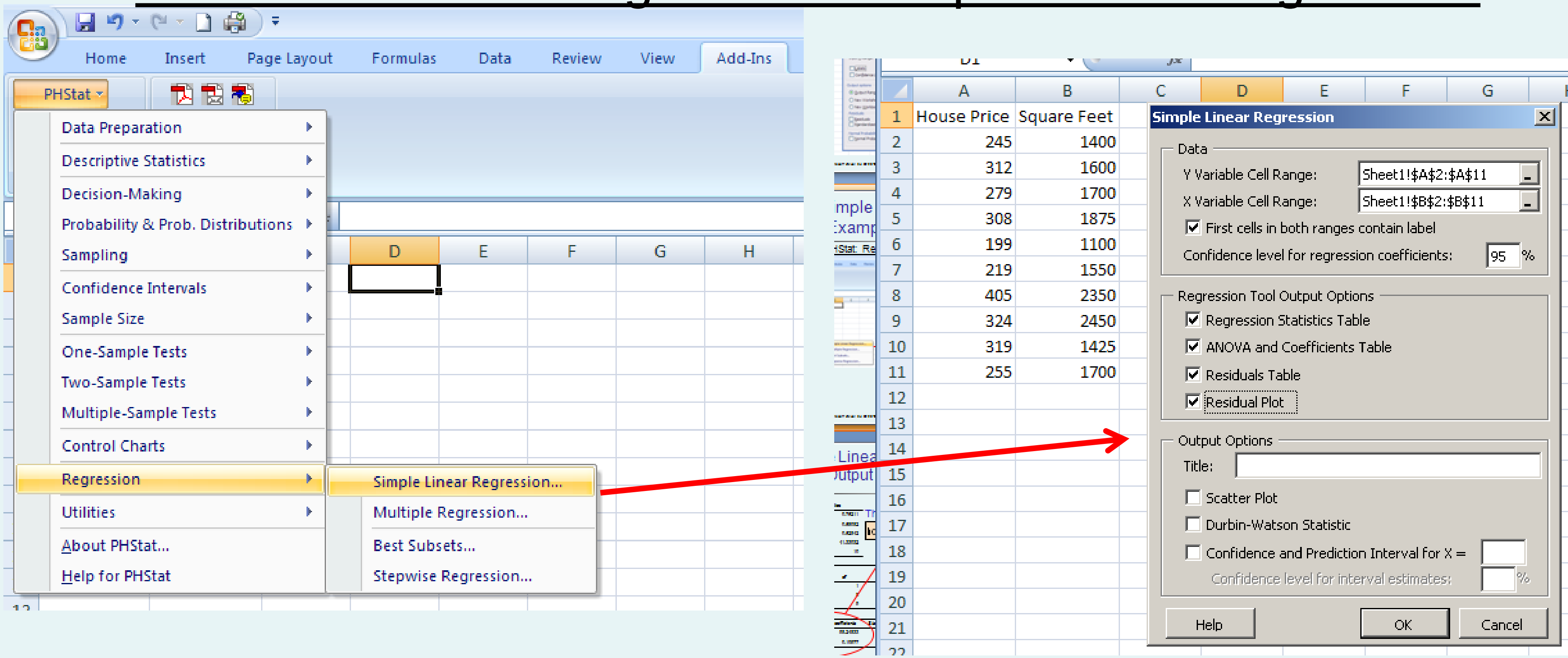


Simple Linear Regression Example: Using PHStat

Add-Ins: PHStat: Regression: Simple Linear Regression



Simple Linear Regression Example: Excel Output

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Regression Statistics	
Multiple R	0.76211
R Square	0.58082
Adjusted R Square	0.52842
Standard Error	41.33032
Observations	10

The regression equation is:

$$\widehat{\text{house price}} = 98.24833 + 0.10977\ (\text{square feet})$$

ANOVA

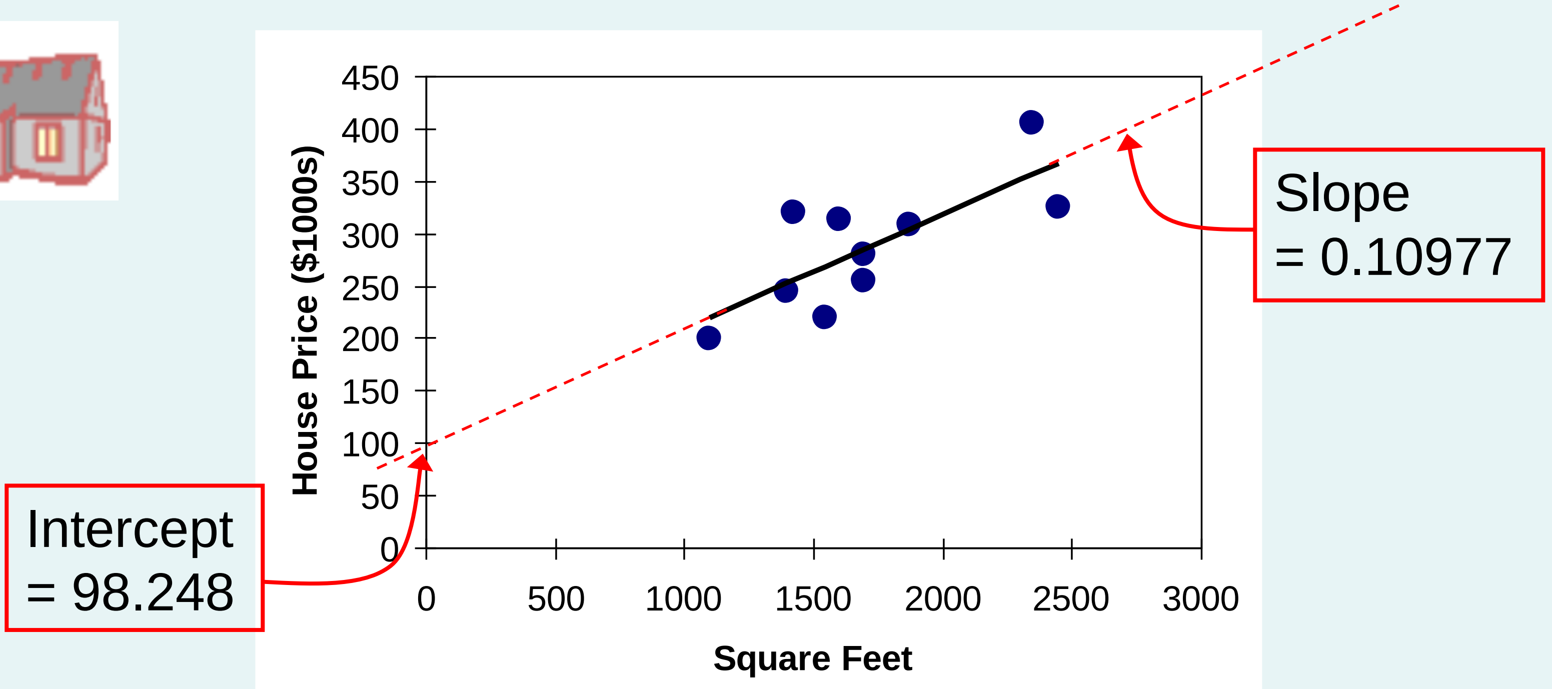
	df	SS	MS	F	Significance F
Regression	1	18934.9348	18934.9348	11.0848	0.01039
Residual	8	13665.5652	1708.1957		
Total	9	32600.5000			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
Intercept	98.24833	58.03348	1.69296	0.12892	-35.57720	232.07386
Square Feet	0.10977	0.03297	3.32938	0.01039	0.03374	0.18580

Simple Linear Regression Example: Graphical Representation

DCOVA

House price model: Scatter Plot and Prediction Line



$$\widehat{\text{house price}} = 98.24833 + 0.10977 \text{ (square feet)}$$

Simple Linear Regression Example: Interpretation of b_o

DCOVA

$$\widehat{\text{house price}} = \boxed{98.24833} + 0.10977\ (\text{square feet})$$

- b_0 is the estimated average value of Y when the value of X is zero (if X = 0 is in the range of observed X values)
- Because a house cannot have a square footage of 0, b_0 has no practical application

Simple Linear Regression Example: Interpreting b_1

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$$\widehat{\text{house price}} = 98.24833 + 0.10977 \text{ (square feet)}$$

- b_1 estimates the change in the average value of Y as a result of a one-unit increase in X
  - Here, $b_1 = 0.10977$ tells us that the mean value of a house increases by .10977($1000) = $109.77, on average, for each additional one square foot of size

Simple Linear Regression Example: Making Predictions

DCOVA

Predict the price for a house with 2000 square feet:

$$\widehat{\text{house price}} = 98.25 + 0.1098\ (\text{sq.ft.})$$

$$= 98.25 + 0.1098(2000)$$

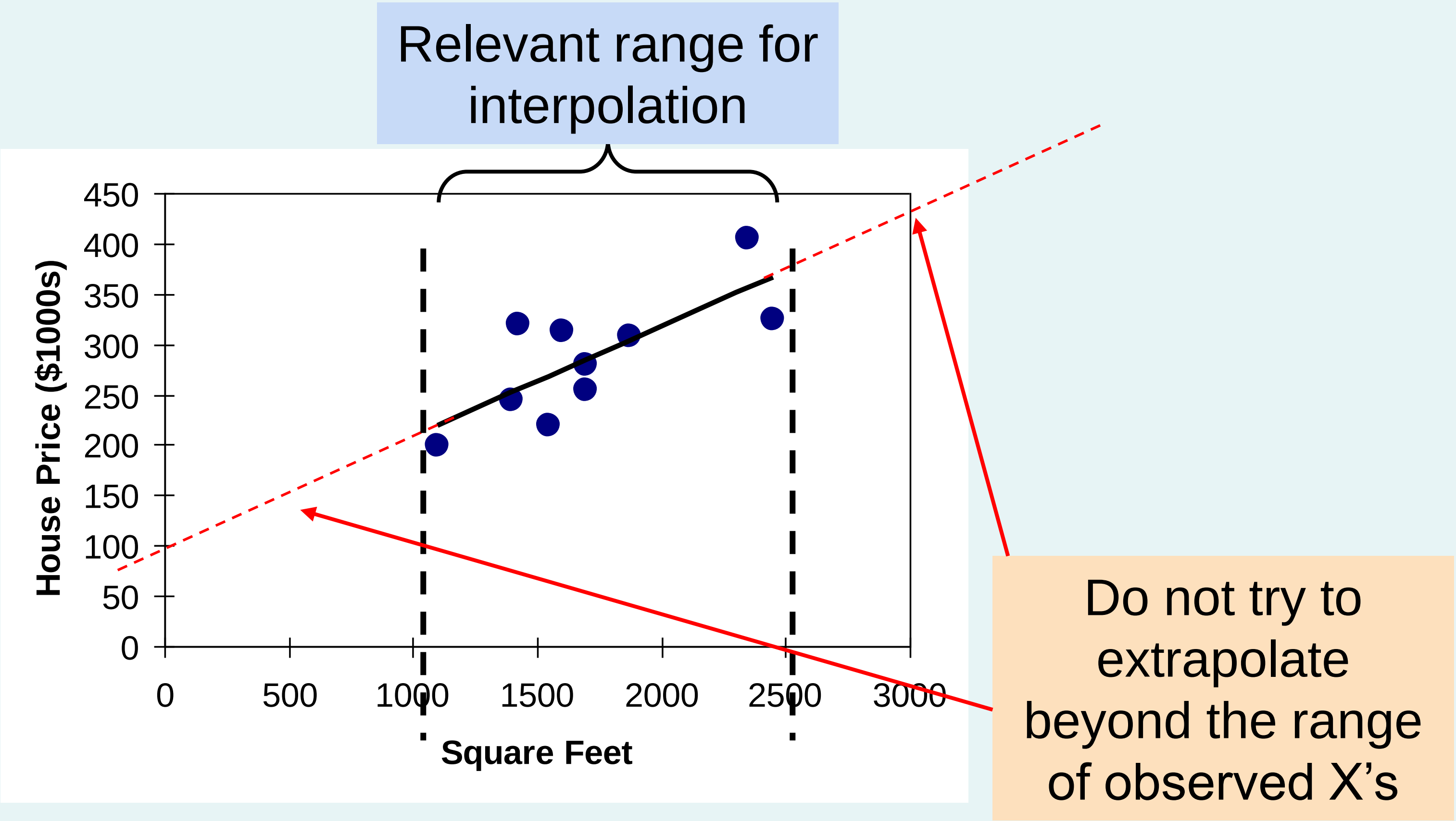
$$= 317.85$$

The predicted price for a house with 2000 square feet is 317.85($1,000s) = $317,850

Simple Linear Regression Example: Making Predictions

DCOVA

- When using a regression model for prediction, only predict within the relevant range of data



Coefficient of Determination, r^2

DCOVA

- The coefficient of determination is the portion of the total variation in the dependent variable that is explained by variation in the independent variable
- The coefficient of determination is also called r-squared and is denoted as r^2

$$r^2 = \frac{SSR}{SST} = \frac{\text{regression sum of squares}}{\text{total sum of squares}}$$

note: $0 \le r^2 \le 1$

Examples of Approximate r^2 Values

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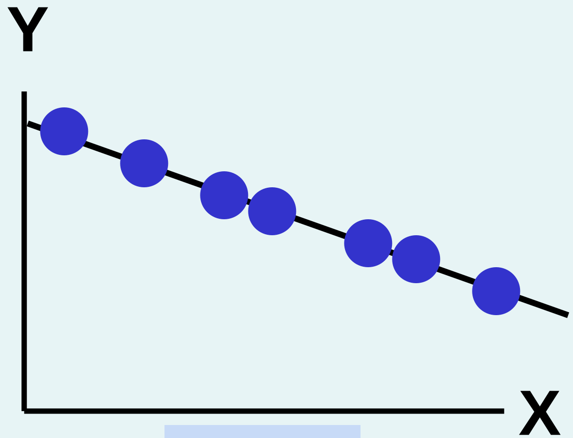
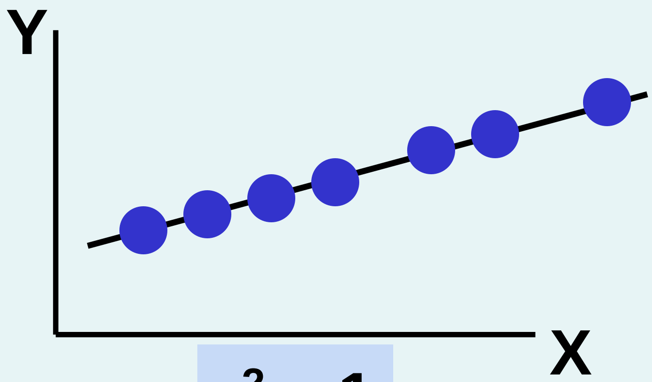


$r^2 = 1$



$r^2 = 1$

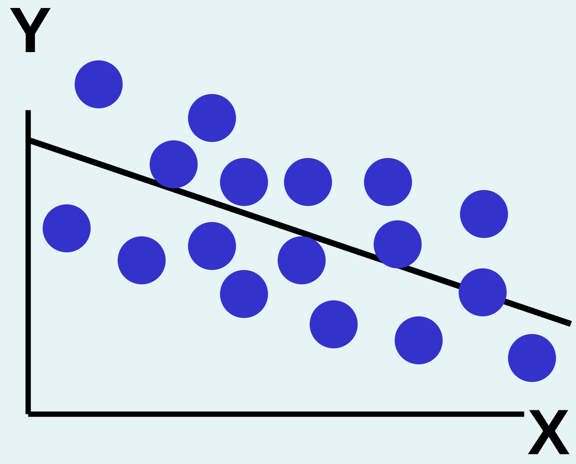
$r^2 = 1$

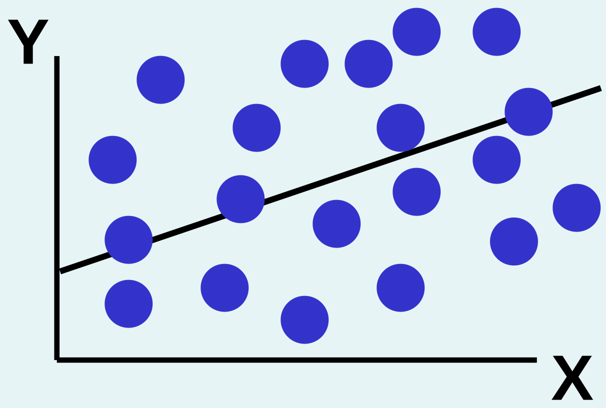
Perfect linear relationship between X and Y:

100% of the variation in Y is explained by variation in X

Examples of Approximate r^2 Values

DCOVA





$0 < r^2 < 1$

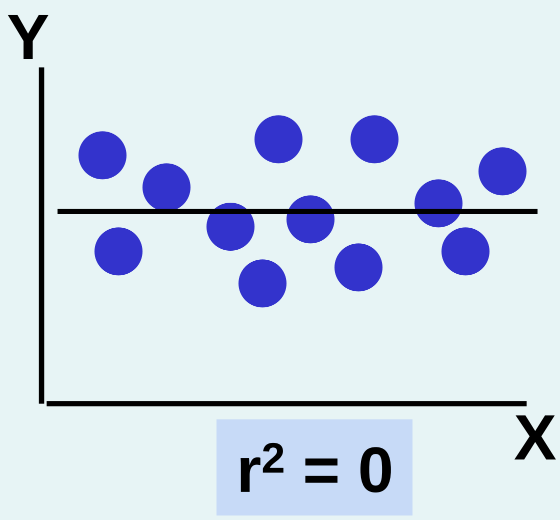
Weaker linear relationships between X and Y:

Some but not all of the variation in Y is explained by variation in X

Examples of Approximate r^2 Values

DCOVA

$r^2 = 0$



No linear relationship between X and Y:

The value of Y does not depend on X. (None of the variation in Y is explained by variation in X)

Simple Linear Regression Example: Coefficient of Determination, r^2 in Excel

DCOVA

Regression Statistics	
Multiple R	0.76211
R Square	0.58082
Adjusted R Square	0.52842
Standard Error	41.33032
Observations	10

$$r^2 = \frac{SSR}{SST} = \frac{18934.9348}{32600.5000} = 0.58082$$

58.08% of the variation in house prices is explained by variation in square feet

ANOVA

	df	SS	MS	F	Significance F
Regression	1	18934.9348	18934.9348	11.0848	0.01039
Residual	8	13665.5652	1708.1957		
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EXAMPLE:

Eight students revealed the number of hours per day spent studying for the final exams and their relevant marks. These are tabulated in a table.

- Find the least square regression equation.
- Interpret the value of *a* and *b*.
- Predict the mark of final exam, if the student spent 9 hours and 15 minutes for studying.

Number of hours	2	6	8	1	10	7	6	3
Marks	40	50	80	20	60	80	90	40

SOLUTION:

$$b = \frac{SS_{xy}}{SS_{xx}} = 5.7090$$

$$a = \bar{y} - b\bar{x} = \frac{\sum y}{n} - b\frac{\sum x}{n} = 26.81$$

$$\hat{y} = 26.81 + 5.7090x$$