CHAPTER 4 : TIME VALUE OF MONEY

KEY CONCEPTS AND SKILLS

❑ Be able to compute:

- The future value of an investment made today

- The present value of cash to be received at some future date

- The return on an investment

- The number of periods that equates a present value and a future value given an interest rate

❑ Be able to solve time value of money problems using:

- Formulas

- Financial Calculator

CHAPTER OUTLINE

1.   Future Value and Compounding

2.   Present Value and Discounting

3.   More on Present and Future Values

Solving for:

Implied interest rate

Number of periods

BASIC DEFINITIONS

❑ **Present Value   (PV)**

- The current value of future cash flows discounted at the appropriate discount rate

- Value at t=0 on a timeline

❑ **Future Value   (FV)**

- The amount an investment is worth after one or more periods.

- "Later" money on a timeline

TIMELINE OF CASH FLOWS

TimeLine for a $100 Lump Sum due at the End of Year 2.

FUTURE VALUES: GENERAL FORMULA

$$FV = PV(1 + r)^t$$

FV = future value

PV = present value

r = period interest rate, expressed as a decimal

t = number of periods

❑ Future value interest factor = $(1 + r)^t$

FUTURE VALUES – EXAMPLE 1

Suppose you invest \$100 for one year at 10% per year. What is the future value in one year?

- Interest = 100(.10) = 10
- Value in one year

= Principal + interest

= 100 + 10 = 110

- Future Value (FV)

= 100(1 + .10) = 110

FUTURE VALUES – EXAMPLE 2

Suppose you leave the money in for another year. How much will you have two years from now?

FV = 100(1.10)(1.10)

= $100(1.10)^2$ = 121.00

EFFECTS OF COMPOUNDING

❑ Simple interest
- Interest earned only on the original principal

❑ Compound interest
- Interest earned on principal and on interest received
- "Interest on interest" – interest earned on reinvestment of previous interest payments

EFFECTS OF COMPOUNDING

❑ **Consider the previous example**

❑ FV w/simple interest

= 100 + 10 + 10 = 120

❑ FV w/compound interest

= $100(1.10)^2$ = 121.00

❑ The extra 1.00 comes from the interest of .10(10) = 1.00 earned on the first interest payment

FUTURE VALUES – EXAMPLE 3

1. **Suppose you invest the $100 from the previous example for 5 years. How much would you have?**
2. **Suppose you had a relative deposit $10 at 5.5% interest 200 years ago. How much would the investment be worth today?**

FUTURE VALUE: GENERAL GROWTH FORMULA

1. **Suppose your company expects to increase unit sales of widgets by 15% per year for the next 5 years. If you currently sell 3 million widgets in one year, how many widgets do you expect to sell in 5 years?**

FUTURE VALUE:
Important Relationship I

For a given interest rate:

❑ The longer the time period,

❑ The higher the future value

$$FV = PV(1 + r)^t$$

FUTURE VALUE
Important Relationship II

For a given time period:

❑ The higher the interest rate,

❑ The larger the future value

FIGURE B

PRESENT VALUES

❑ The current value of future cash flows discounted at the appropriate discount rate

❑ Value at t=0 on a timeline

❑ Answers the questions:

▪ **How much do I have to invest today to have some amount in the future?**

▪ **What is the current value of an amount to be received in the future?**

PRESENT VALUES

❑ Present Value = the current value of an amount to be received in the future

❑ Why is it worth less than face value?

   ▪ Opportunity cost

   ▪ Risk & Uncertainty

   ▪ Discount Rate = ƒ (time, risk)

TIMELINE OF CASH FLOWS

PRESENT VALUES

$$FV = PV(1 + r)^t$$

❑ Rearrange to solve for PV

$$PV = FV / (1+r)^t$$

$$PV = FV(1+r)^{-t}$$

❑ "Discounting" = finding the present value of one or more future amounts.

What's the PV of $100 due in 3 Years if r = 10%?

PRESENT VALUE: EXAMPLE 1 (SINGLE PERIOD)

1. **Suppose you need $10,000 in one year for the down payment on a new car. If you can earn 7% annually, how much do you need to invest today?**

PRESENT VALUES: EXAMPLE 2 (MULTI-PERIODS)

1. **You want to begin saving for your daughter's college education and you estimate that she will need $150,000 in 17 years. If you feel confident that you can earn 8% per year, how much do you need to invest today?**

PRESENT VALUES: EXAMPLE 3 (MULTI-PERIODS)

1. **Your parents set up a trust fund for you 10 years ago that is now worth $19,671.51. If the fund earned 7% per year, how much did your parents invest?**

PRESENT VALUE:
Important Relationship I

For a given interest rate:

   ❑ The longer the time period,

   ❑ The lower the present value

PRESENT VALUE: IMPORTANT RELATIONSHIP I

1. **What is the present value of $500 to be received in 5 years? 10 years? The discount rate is 10%**

PRESENT VALUE
Important Relationship II

For a given time period:

❏ The higher the interest rate,

❏ The smaller the present value

PRESENT VALUE: IMPORTANT RELATIONSHIP II

1. **What is the present value of $500 received in 5 years if the interest rate is 10%? 15%?**

DISCOUNT RATE

To find the implied interest rate, rearrange the basic PV equation and solve for r:

$$FV = PV(1 + r)^t$$

$$r = (FV / PV)^{1/t} – 1$$

DISCOUNT RATE – EXAMPLE

1. **You are looking at an investment that will pay $1200 in 5 years if you invest $1000 today. What is the implied rate of interest?**

Formula:

$r = (1200 / 1000)^{1/5} – 1 = .03714 = 3.714\%$

FINDING THE NUMBER OF PERIODS

Start with basic equation and solve for t:

$$FV = PV(1 + r)^t$$

NUMBER OF PERIODS – EXAMPLE

1. **You want to purchase a new car and you are willing to pay $20,000. If you can invest at 10% per year and you currently have $15,000, how long will it be before you have enough money to pay cash for the car?**

=3.02

NUMBER OF PERIODS – EXAMPLE

❏ Formula Solution:

- FV/PV = 20,000/15,000 = 1.333

- ln(1.333) = 0.2877

- ln(1.10) = 0.0953

- t = 0.2877/0.0953 = 3.0189